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# Moving jammer suppression with robust blind adaptive algorithms in GPS receiver<sup>\*</sup>

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**Abstract** The problem of providing robustness to the conventional narrow-band uniform linear array configuration so as to handle moving and cochannel jammers in GPS receiver is addressed here. This robustness is obtained via the use of derivative constraints in jammer directions. We develop the robust power inversion (PI) algorithm, the loading robust PI algorithm, and the robust eigenvector projection (EP) algorithm. These three algorithms are characterized by the fact that there is no need to know the directions of the GPS signals and jammers. Simulation results show that the algorithms proposed here are suitable for dealing with moving or cochannel jammers.

**Key words** GPS; moving jammer; interference suppression

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## 稳健的 GPS 动态干扰盲自适应抑制算法

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**摘要** 研究当动态干扰或同信道干扰存在时,如何提高抗干扰 GPS 接收机的稳健性问题. 在干扰来向上,通过增加微分约束条件,可以将零陷展宽,从而提高其稳健性. 主要研究 3 种稳健自适应抗干扰算法:稳健的功率倒置(PI)算法、加载的稳健功率倒置算法和稳健特征向量投影(EP)算法. 3 种算法均不需要预知 GPS 信号和干扰信号的来向等先验信息. 仿真结果表明,以上 3 种稳健算法适用于当干扰源是运动的或同信道干扰源的情形.

**关键词** GPS; 动态干扰; 干扰抑制

A global positioning system receiver computes its position, velocity, and time solution by processing navigation signals from a constellation of GPS satellites. These signals arrive at the receiver at

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a very low power level, typically 20-30 dB below the receiver's thermal noise level. If the power of the interference exceeds the processing gain offered via the spreading/dispreading of the GPS C/A code, the GPS receiver will lose the ability to navigate<sup>[1-3]</sup>. Modern GPS receivers usually use antenna array to produce a digital wave beam. Thus adaptive algorithms can be used to produce a digital wave beam, which can result in nulling steering in interference direction, eliminate interference effect and suppress the interference by getting the lower gain. There is a large body of literature on such optimum beamforming techniques<sup>[4-5]</sup>. The performance of such optimum beamformers, however, degrades significantly in the presence of moving jammers. The robust algorithms against fast moving jammers were considered via the use of certain derivative constraints in the jamming directions<sup>[6-7]</sup>, however, the desired signal directions have to be known as a priori information in such solutions. Unfortunately, the knowledge of GPS signal directions is not simple or requires additional complexity. In this study, we propose novel interference suppression schemes, where the robustness property is incorporated into the adaptive algorithms, and especially, any priori information on the directions jammers and desired GPS signals is not required.

## 1 Signal model

We formulate the problem for a uniform linear array of  $M$  sensors taking into account that the approach considered is applicable to planar or volume arrays as well. Let the  $L$  desired GPS signals impinge on the array from directions  $\{\theta_{s_1}, \theta_{s_2}, \dots, \theta_{s_L}\}$  along with  $P$  jammer signals from unknown directions  $\{\theta_{j_1}, \theta_{j_2}, \dots, \theta_{j_P}\}$ , respectively. Let the jammers be uncorrelated with each other as well as with the signals. Therefore, the output vector of the array at the  $t$ th time snapshot

$$y(t) = A_s s(t) + A_j j(t) + n(t), \quad t = 0, 1, 2, \dots, \quad (1)$$

where  $s(t) = [s_1(t), s_2(t), \dots, s_L(t)]^T$  is the  $L \times 1$  signal waveforms of the desired GPS signals and  $A_s = [a(\theta_{s_1}), a(\theta_{s_2}), \dots, a(\theta_{s_L})]^T$  is the corresponding  $M \times L$  steering vector, Here  $(\cdot)^T$  denotes the transpose. The desired GPS signal vector  $a(\theta_{s_i})$  can be modeled as plane waves:

$$a(\theta_{s_i}) = (\exp\{jx_1\xi\}, \exp\{jx_2\xi\}, \dots, \exp\{jx_M\xi\})^T \quad (2)$$

where  $\xi = (2\pi/\lambda)\sin\theta$ ,  $\lambda$  is the wavelength,  $x_l = \sum_{i=1}^{l-1} d_{i,i+1}$  is the coordinate of the  $l$ th sensor,  $d_{i,i+1}$  is the distance between the  $i$ th and  $(i+1)$ th sensors,  $j(t) = [j_1(t), j_2(t), \dots, j_P(t)]^T$  is the vector of narrowband jammer waveforms, and  $A_j = [a(\theta_{j_1}), a(\theta_{j_2}), \dots, a(\theta_{j_P})]^T$ . The  $M \times 1$  vector  $n(t)$  contains the random white sensors noise.

## 2 Robust algorithms with derivative constraints

Under the assumption that the GPS signals, interferences, and noise are independent, the covariance matrix of the received data can be expressed as

$$\begin{aligned} R &= E[XX^H] = R_s + R_j + R_v \\ &= A_s S A_s^H + A_j J A_j^H + \sigma^2 I, \end{aligned} \quad (3)$$

where  $E\{\cdot\}$  denotes the statistical expectation,  $R_s$ ,  $R_j$ , and  $R_v$  are the covariance matrices of the GPS signals, interferences, and noise, respectively,  $I$  is the  $M \times M$  identity matrix,  $\sigma^2$  is the power of the noise, and  $(\cdot)^H$  denotes the Hermitian transpose. Typically, the jammers are much more powerful than the sensor noises and the GPS signals, furthermore, the desired GPS signals are well below the noise floor (typically 20-30 dB below the noise floor). Therefore, the total received signal power is mainly dominated by the interference signals and the noise. We use this information in the following.

Thus the covariance matrix  $R$  can be simplified to

$$R \approx R_j + \sigma^2 I = A_j J A_j^H + \sigma^2 I. \quad (4)$$

Applying the matrix inversion lemma to the matrix  $R$ , we obtain

$$\frac{1}{\sigma^2}\mathbf{R} = \mathbf{I} + \frac{1}{\sigma^2}(\mathbf{A}_j\mathbf{J}\mathbf{A}_j). \tag{5}$$

With the property in Ref. [7], we obtain

$$\begin{aligned} \lim_{\frac{1}{\sigma^2} \rightarrow \infty} \frac{1}{\sigma^2} \mathbf{R}^{-1} &= \lim_{\frac{1}{\sigma^2} \rightarrow \infty} \left[ \mathbf{I} + \frac{1}{\sigma^2}(\mathbf{A}_j\mathbf{J}\mathbf{A}_j) \right]^{-1} \\ &= \mathbf{P}_A^\perp = \mathbf{I} - \mathbf{P}_A, \end{aligned} \tag{6}$$

where  $\mathbf{P}_A$  is the orthogonal projection onto the jammer subspace. The solution for the optimum weight vector of the adaptive array maximizing the signal-to-interference-plus-noise ratio ( SINR ) can be obtained from

$$\boldsymbol{\omega}_{\text{opt}} = \mu \mathbf{R}^{-1} \mathbf{a}(\theta_{s_i}), \quad i = 1, \cdots, L, \tag{7}$$

where  $\mu$  is a constant, which does not affect the output SINR and will be omitted in the tolluing without loss of generality. For strong jammers, the optimal weight vector in Eq. (7) tends to be orthogonal to the jammer subspace  $\{ \mathbf{a}(\theta_{j_1}), \mathbf{a}(\theta_{j_2}), \cdots, \mathbf{a}(\theta_{j_p}) \}$ . This means that the array pattern  $\boldsymbol{\omega}_{\text{opt}}^H \mathbf{a}(\theta)$  has nulls in the directions of arrival (DOA) of the jammers, as well as main beams in the desired GPS signal directions.

Taking a cue from the power inversion (PI) algorithm, we can rewrite Eq. (7) as follows:

$$\boldsymbol{\omega}_{\text{opt}} = \mathbf{R}^{-1} \mathbf{s}_0, \tag{8}$$

where  $\mathbf{s}_0 = [1, 0, 0, \cdots, 0]^T$  is an  $M \times 1$  vector. From Eq. (8) we can deduce that a priori knowledge of the GPS signal directions is not required. To broaden the width of the pattern nulls in the jammer directions, we can introduce  $p$ th-order derivative constraints

$$\begin{aligned} \left. \frac{\partial (\boldsymbol{\omega}^H \mathbf{a}(\theta))}{\partial \xi^m} \right|_{\theta = \theta_k} &= 0, \\ k &= 1, 2, \cdots, P; \quad m = 1, 2, \cdots, p. \end{aligned} \tag{9}$$

For high interference-to-noise ratio (JNR) in an GPS receiver, a projection orthogonal to the array data is the same as a projection orthogonal to the jammer subspace. Based on the above analysis, we obtain  $\mathbf{P}_Y = \mathbf{P}_A$ , where  $\mathbf{Y}$  denotes the  $M \times N$  data matrix consisting of  $N$  stationary independent snapshot vector  $\mathbf{y}(t)$  in the columns.

2.1 Robust PI algorithm

Typically, the directions of the desired signals and jammers are unknown in a moving jammer

situation, thus a blind adaptive jammer suppression algorithm can be more applicable. The robust PI algorithm calculates the weight vector by using the estimation of the sampling covariance matrix and only the first-order constraint is considered here, although any order of constraints is possible. Thus the robust PI algorithm can be expressed as

$$\begin{aligned} \hat{\mathbf{R}}(k) &= (\mathbf{Y}(k)\mathbf{Y}(k)^H + \zeta \mathbf{B}\mathbf{Y}(k)\mathbf{Y}(k)^H \mathbf{B})/N, \\ \mathbf{B} &= \text{diag}\{x_1, x_2, \cdots, x_n\}, \\ \mathbf{Y}(k) &= [y(k+1), y(k+2), \cdots, y(k+N)]^T, \end{aligned} \tag{10}$$

$$\boldsymbol{\omega}_{\text{opt}} = \hat{\mathbf{R}}^{-1} \mathbf{s}_0, \tag{12}$$

where  $N$  is the number of the snapshots,  $\zeta$  is a real positive weight which controls the relative contribution of the “derivative” data. The choice of the parameter  $\zeta$  is very important for the optimization of the adaptive array performance. If the contribution of the “derivative” data vectors is too strong as compared with the contribution of the original data, the depth of the nulls is not sufficient, and as a result, the jamming power is not sufficiently suppressed. Otherwise, when the contribution of the original data is much stronger than that of the “derivative” data, the desired null width may not be sufficient. The value of  $\zeta$  from the compromise between the null depth and width of the adapted pattern is presented in Ref. [7].

2.2 Loaded robust PI algorithm

In the loading robust PI algorithm, a small real positive number  $\gamma$  is added to the diagonal of the covariance matrix, which is a simple and efficient method to improve the robustness of adaptive beamforming for the cases due to the fast moving interference. Recently, numerous studies have been devoted to finding the data dependent method for determining the diagonal loading value<sup>[8-10]</sup>. The larger the covariance matrix estimation error is, the larger the diagonal loading value will be. By this fact, one can expect a better performance of the loaded robust PI method for small number of snapshots than that of the conventional robust PI method. Here we formulate the loading robust PI

algorithm as

$$\begin{aligned}\hat{\mathbf{R}}_{dl}(k) &= \gamma \mathbf{I} + \hat{\mathbf{R}}(k) \\ &= \gamma \mathbf{I} + (\mathbf{Y}(k)\mathbf{Y}(k)^H + \zeta \mathbf{B}\mathbf{Y}(k)\mathbf{Y}(k)^H \mathbf{B})/N,\end{aligned}\quad (13)$$

$$\boldsymbol{\omega}_{\text{opt}} = \mu \hat{\mathbf{R}}_{dl}^{-1} \mathbf{s}_0, \quad (14)$$

where we consider first-order constraints, i. e.,  $p = 1$ .

### 2.3 Robust eigenvector projection (EP) algorithm

Several subspace projection techniques are available for adaptive beamforming<sup>[11-12]</sup>, and in this work, we use the eigenvalue decomposition algorithm. With the approximation of Eq. (4), the covariance matrix  $\mathbf{R}$  can be decomposed into two subspaces

$$\begin{aligned}\mathbf{R} &= \sum_{m=1}^M \lambda_m \mathbf{e}_m \mathbf{e}_m^H \approx \sum_{m=1}^P \lambda_m \mathbf{e}_m \mathbf{e}_m^H + \sigma^2 \sum_{m=P+1}^M \lambda_m \mathbf{e}_m \mathbf{e}_m^H \\ &= \sum_{m=1}^P \lambda_m \mathbf{e}_m \mathbf{e}_m^H + \sigma^2 \sum_{m=P+1}^M \mathbf{e}_m \mathbf{e}_m^H,\end{aligned}\quad (15)$$

where  $\sigma^2$  denotes the power of the white noise,  $\lambda_m (m = 1, \dots, M)$ ,  $\lambda_1 > \lambda_2 > \dots > \lambda_M$  are the ordered sample eigenvalues. Of these,  $\lambda_m (m = 1, \dots, P)$  are the  $P$  dominant eigenvalues, and the associated eigenvectors  $\mathbf{e}_m (m = 1, \dots, P)$  span the jammer subspace  $\mathbf{U}_j$ . The eigenvectors, which associated with the rest  $M - P$  eigenvalues  $\lambda_m (m = P + 1, \dots, M)$ , span the noise subspace  $\mathbf{U}_n$ . Then, the jammer subspace can be formulated as

$$\mathbf{U}_j = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_P]. \quad (16)$$

Once the jammer subspace is available, the noise subspace can be obtained from the orthogonal projection of the jammer subspace, which is given by

$$\mathbf{U}_j^\perp = \mathbf{I} - \mathbf{U}_j (\mathbf{U}_j^H \mathbf{U}_j)^{-1} \mathbf{U}_j^H. \quad (17)$$

In the robust EP algorithm, it is useful to choose the largest eigenvector of  $\mathbf{Q}$  (typically,  $\mathbf{Q} \geq \mathbf{P}$ ) to span the jammer subspace due to the moving jammer case. Following these considerations, the solution of the weight vector is now given by

$$\boldsymbol{\omega}_{\text{opt}} = \mathbf{P}_V^\perp \mathbf{s}_0, \quad (18a)$$

it follows that

$$\mathbf{P}_V^\perp = \mathbf{I} - \mathbf{V}(\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H, \quad (18b)$$

where

$$\mathbf{V} = [\mathbf{U}_j, \mathbf{B}\mathbf{U}_j, \dots, \mathbf{B}^p \mathbf{U}_j], \quad (18c)$$

and

$$\mathbf{U}_j = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_P]. \quad (18d)$$

Therefore, it is distinct that the robust EP algorithm requires the additional degrees of freedom. In addition, the choice of the optimum  $\mathbf{Q}$  depends not only on the order  $p$  of the derivative constraints, but also on the variation of the jammer directions in the processing snapshots.

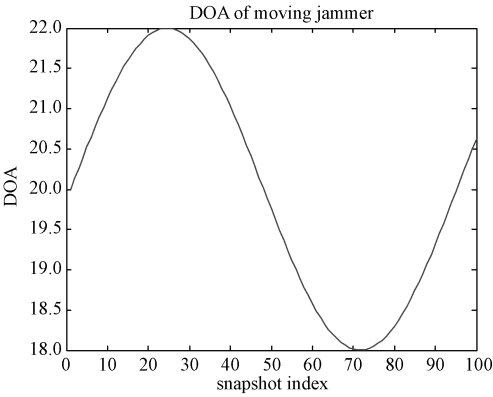
## 3 Simulation and results

Simulation studies have been carried out in this section. For ease of comparison of our algorithms, several scenarios discussed in the sequel have been chosen for this study. In each of these cases, ambient noise at each sensor is assumed to be spatially uncorrelated and has a reference power level of unity. A uniform linear array of 7 elements with half-wavelength space is considered. The beginning of the  $x$ -axis is chosen in the first sensor. Thus from Eq. (10) it follows that

$$\mathbf{B} = \text{diag}\{1, 2, \dots, M-1\}, \quad (19)$$

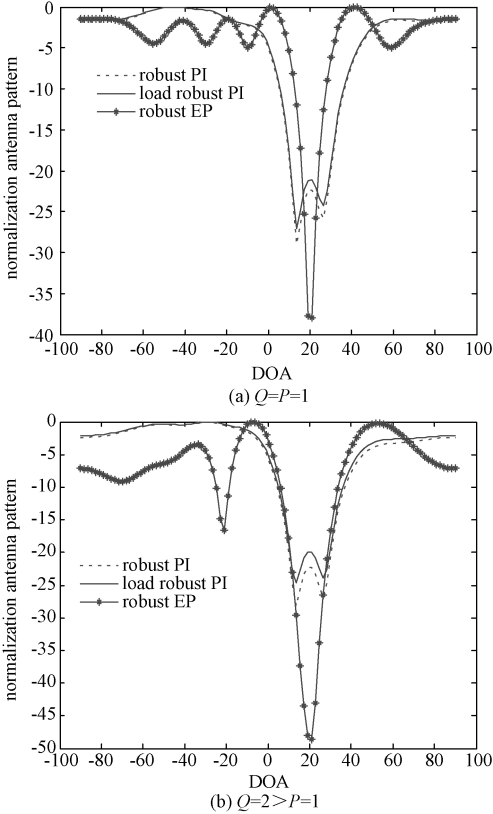
where the common multiplier  $\lambda/2$  can be omitted. Assuming that four GPS signals impinge on the array, numbers 2, 13, 20 and 22, with the incident angle  $\{\theta_{2\#} = 10^\circ, \theta_{13\#} = -30^\circ, \theta_{20\#} = 30^\circ, \theta_{22\#} = 40^\circ\}$ , respectively. There is one cochannel narrowband jammer and the corresponding simulated trajectory of angular jammer motion is  $\theta_{j_1}(i) = 20^\circ + 2^\circ \sin(i/15)$ , where  $i$  denotes the  $i$ th snapshot. Whereafter, the graph of the jammer direction changing versus snapshots is showed in Fig. 1.

The GPS receiver intermediate frequency (IF) is at 4.309 MHz, then the mixed signals, which consist of the GPS signal, jammer and noise are digitized at 6 MHz at the same time. We assume INR = 20 dB for the jammer and SNR = -20 dB for each GPS signal in the sensor, respectively. Figure 2 shows the typical adaptive array patterns corresponding to the different algorithms. This corresponds to the scenario with one moving narrowband jammer sources impinging from direction  $\theta_j = 20^\circ$ . In the loading robust PI, we take  $\gamma =$



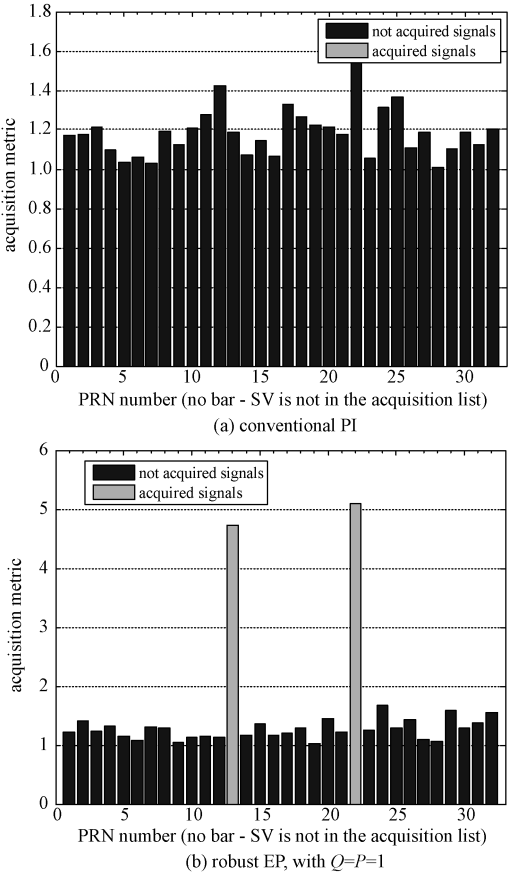
**Fig. 1** Curve of jammer direction changing versus time (snapshot)

$2\sigma^2$ . First-order-constraint is considered for the three robust algorithms. From which the effect of flattening the adaptive nulls can be observed clearly when the number of the eigenvectors increases for the robust EP algorithm,  $Q = 1$  (Fig. 1 (a)) and  $Q = 2$  (Fig. 2 (b)). Of all the algorithms considered here, the directional pattern nulls of the robust EP algorithm are the deepest and the most narrowest when  $Q = P$  is taken in (18).



**Fig. 2** Patterns of 7-element adaptive array by using the different algorithms

Typically, to track and decode the information in the GPS signal, an acquisition method must be used to detect the presence of the GPS signal. In the following, we will investigate the proposed receiver's acquisition capability. Generally, the acquisition can be achieved by cross-correlating the received signal with the locally generated C/A-code<sup>[2]</sup>. Once the receiver captures the satellite, there is a maximum correlation. Figure 3 shows the corresponding acquisition results by using different algorithms available. It is demonstrated that with the conventional PI algorithm, the acquisition fails. If the robust adaptive algorithms proposed are applied, receivers are able to capture the satellites. However, we can also see that two satellites are captured in Fig. 3 (b), while only one satellite is captured in Fig. 3 (c) and Fig. 3 (d). The main reason is that the nulls broadening may lead to the cancellation effect of the signals adjacent to the jammer direction.



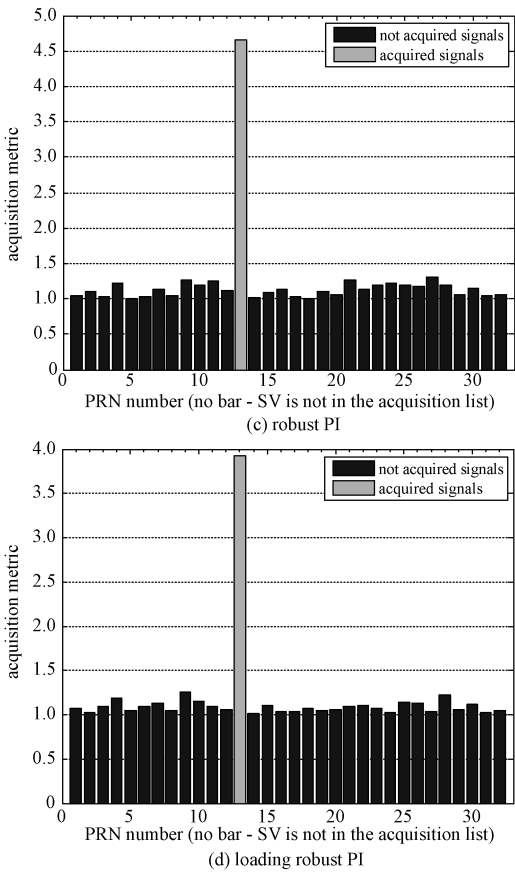


Fig. 3 Satellite signal acquisition results by using the different adaptive algorithms

4 Conclusion

We have presented the novel constrained version of the PI, LPI and EP adaptive algorithms for interference suppressing in an GPS receiver. The main idea of the derivative constrain is to broaden the width of the pattern nulls in the jammer directions to provide more robustness in situations of moving jammers. The detailed computer simulations verify that the proposed robust adaptive algorithms are capable of suppressing the moving jammers without any priori directions information of the jammers and desired GPS signals.

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